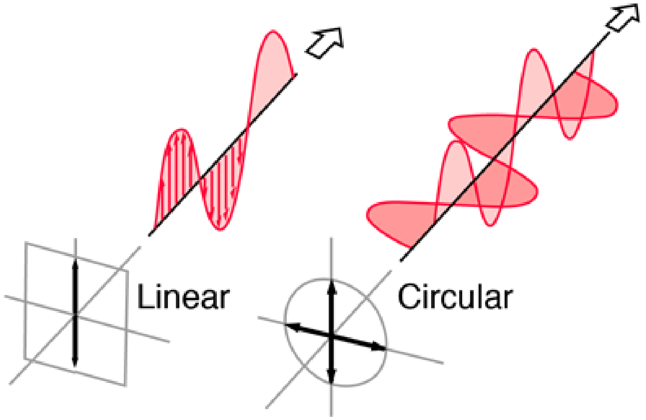
Wave Polarization

- TEM plane waves: Transverse Electro-Magnetic
  - Fields restricted to the plane perpendicular to the propagation direction
- General case: electric-field vector describes an ellipse over time in the perpendicular plane



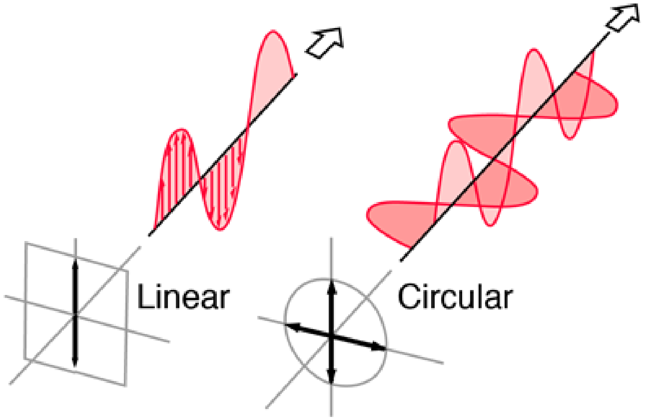
Wave Polarization

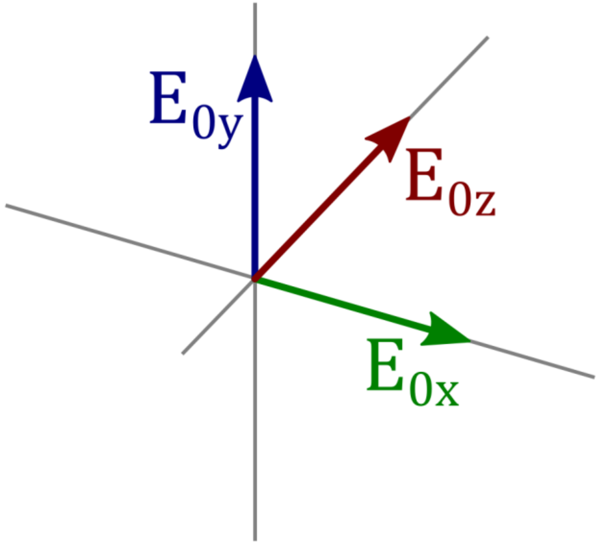
- TEM plane waves: Transverse Electro-Magnetic
  - Fields restricted to the plane perpendicular to the propagation direction
- General case: electric-field vector describes an ellipse over time in the perpendicular plane
- Special cases:
- Linear polarization – Field components E_{0x} and E_{0y} in-phase
- Circular polarization – Field components E_{0x} and E_{0y}:

1) Equal in magnitude

2) Perpendicular

3) Phase-delayed by 90°





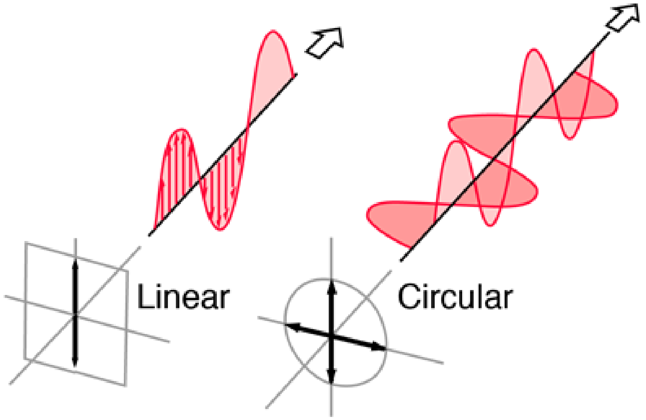
Wave Polarization

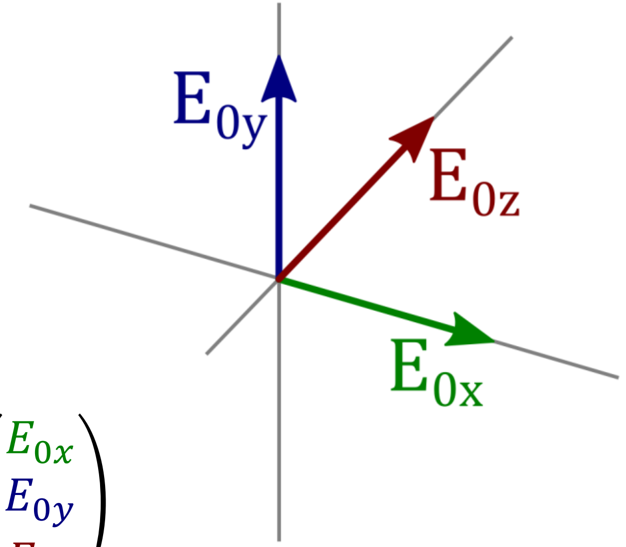
- TEM plane waves: Transverse Electro-Magnetic
  - Fields restricted to the plane perpendicular to the propagation direction
- General case: electric-field vector describes an ellipse over time in the perpendicular plane
- Special cases:
- Linear polarization – Field components E_{0x} and E_{0y} in-phase
- Circular polarization – Field components E_{0x} and E_{0y}:

1) Equal in magnitude

2) Perpendicular

3) Phase-delayed by 90°





$$\boldsymbol{e} = \boldsymbol{pol} = \frac{1}{|E_{tot}|}\begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix}$$

Propagation in z-axis direction: $E_{0z} = 0$

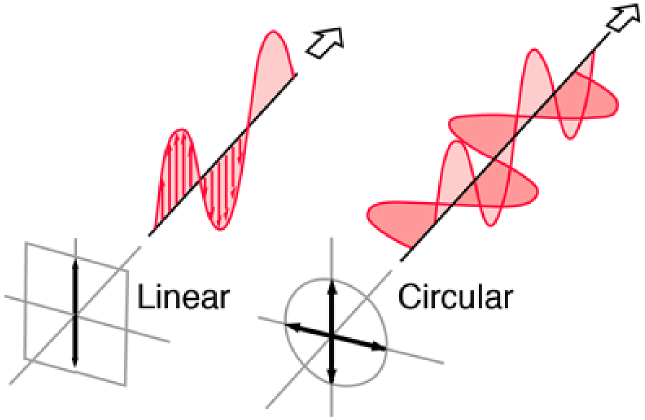
Wave Polarization

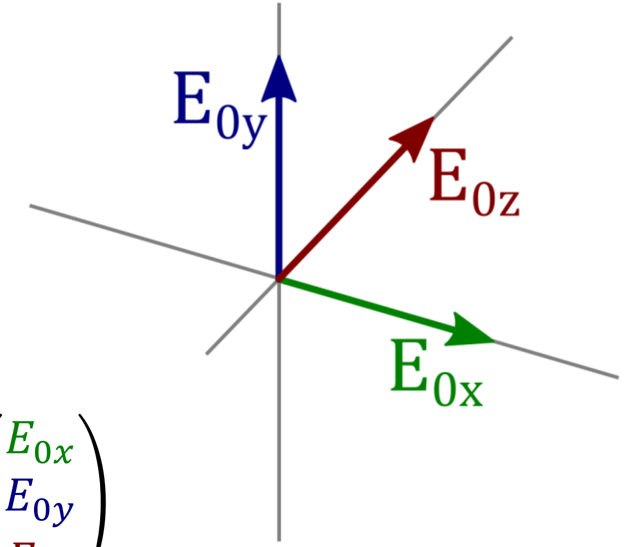
- TEM plane waves: Transverse Electro-Magnetic
  - Fields restricted to the plane perpendicular to the propagation direction
- General case: electric-field vector describes an ellipse over time in the perpendicular plane
- Special cases:
- Linear polarization – Field components E_{0x} and E_{0y} in-phase
- Circular polarization – Field components E_{0x} and E_{0y}:

  1) Equal in magnitude

  2) Perpendicular

  3) Phase-delayed by 90°
- Antennas should have matching polarizations





$$\boldsymbol{e} = \boldsymbol{pol} = \frac{1}{|E_{tot}|}\begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix}$$

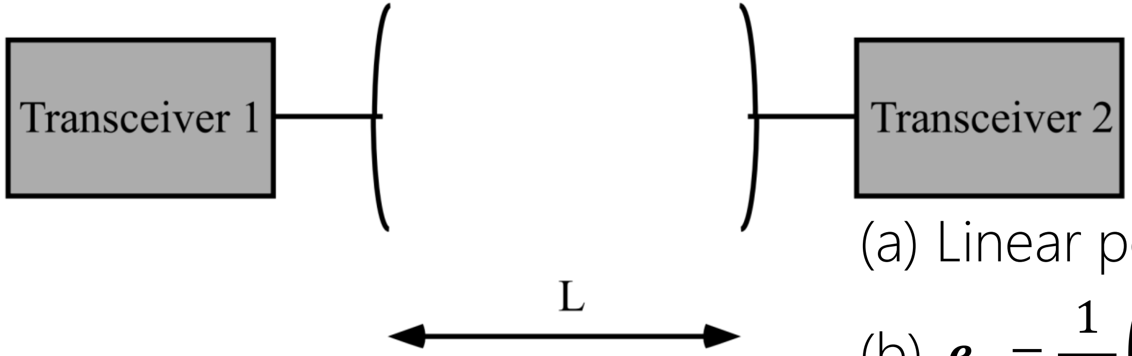
Propagation in z-axis direction: $E_{0z} = 0$

https://www.youtube.com/watch?v=7cNOVOF9FyU

S11E01

- Depolarization factor?

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

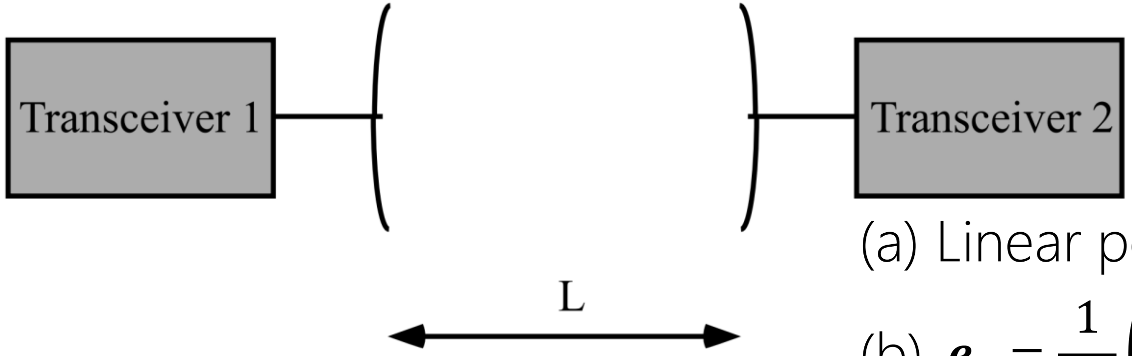
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

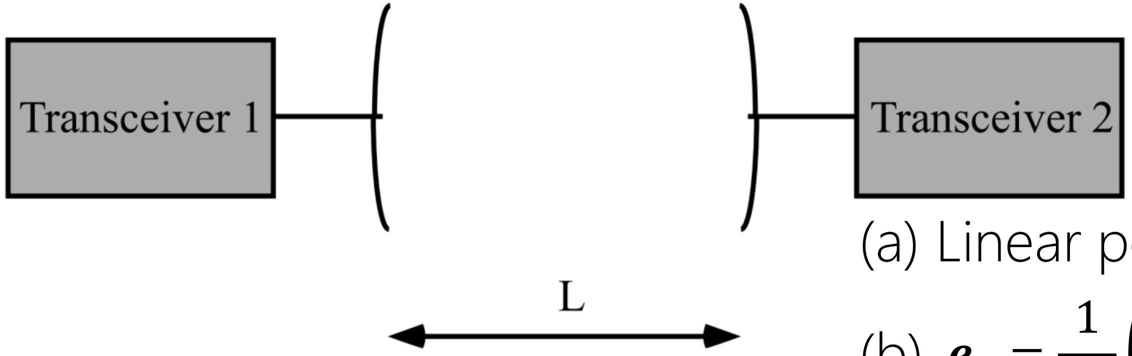
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

(a) Tx/Rx #2 linearly polarized options: $\boldsymbol{e}_2^{(1)} = \frac{1}{x}\begin{pmatrix}x\\ 0\\ 0\end{pmatrix}$ $\quad \boldsymbol{e}_2^{(2)} = \frac{1}{y}\begin{pmatrix}0\\ y\\ 0\end{pmatrix}$

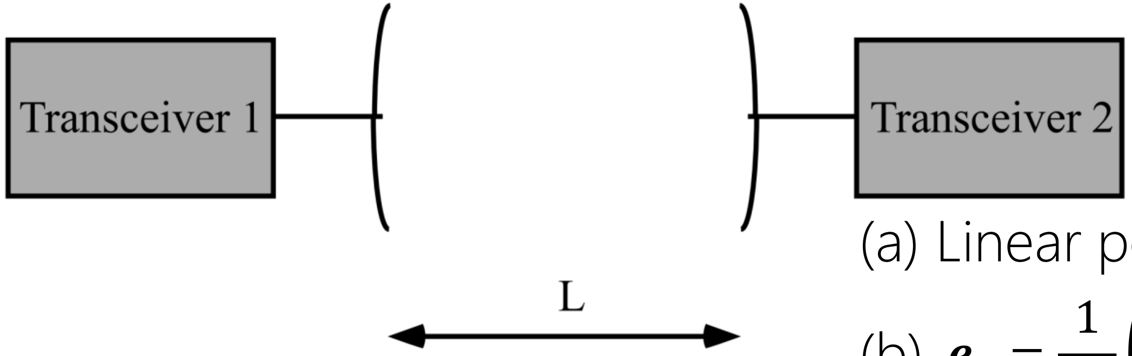
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

(a) Tx/Rx #2 linearly polarized options:

$$\boldsymbol{e}_2^{(1)} = \frac{1}{x}\begin{pmatrix}x\\ 0\\ 0\end{pmatrix} \qquad \boldsymbol{e}_2^{(2)} = \frac{1}{y}\begin{pmatrix}0\\ y\\ 0\end{pmatrix} \qquad \boxed{\boldsymbol{e}_2 = \frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}x\\ y\\ 0\end{pmatrix}}$$

Arbitrary orientation – tilted, but still linear!

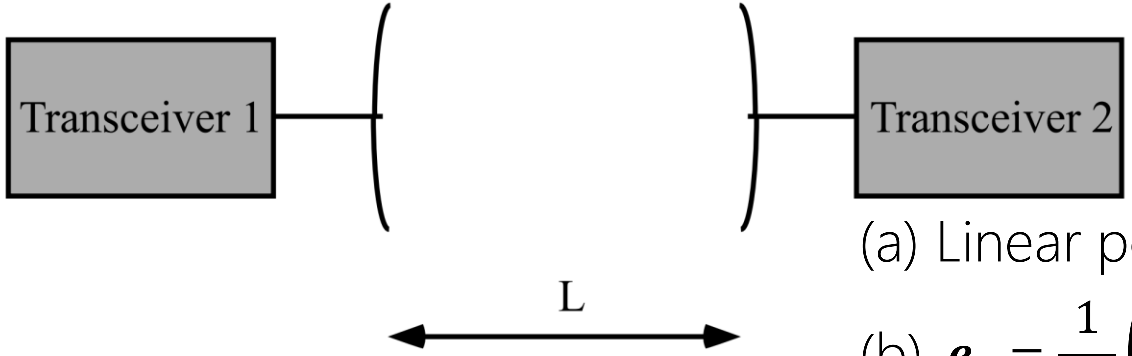
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\j\\0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1+j\\0\end{pmatrix}$

(a) Tx/Rx #2 linearly polarized options:

$$\boldsymbol{e}_2^{(1)} = \frac{1}{x}\begin{pmatrix}x\\0\\0\end{pmatrix} \qquad \boldsymbol{e}_2^{(2)} = \frac{1}{y}\begin{pmatrix}0\\y\\0\end{pmatrix} \qquad \boxed{\boldsymbol{e}_2 = \frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}x\\y\\0\end{pmatrix}}$$

Arbitrary orientation – tilted, but still linear!

$$\chi_{pol}^{(a)} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2 = \left|\frac{1}{\sqrt{2}}\frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}1\\j\\0\end{pmatrix}\cdot\begin{pmatrix}x\\y\\0\end{pmatrix}\right|^2 = \frac{1}{2(x^2+y^2)}|x+jy+0|^2 = \frac{x^2+y^2}{2(x^2+y^2)} = 0.5$$

$$\chi_{pol,dB}^{(a)} = 10\log_{10} 0.5 = -3 \text{ dB}$$

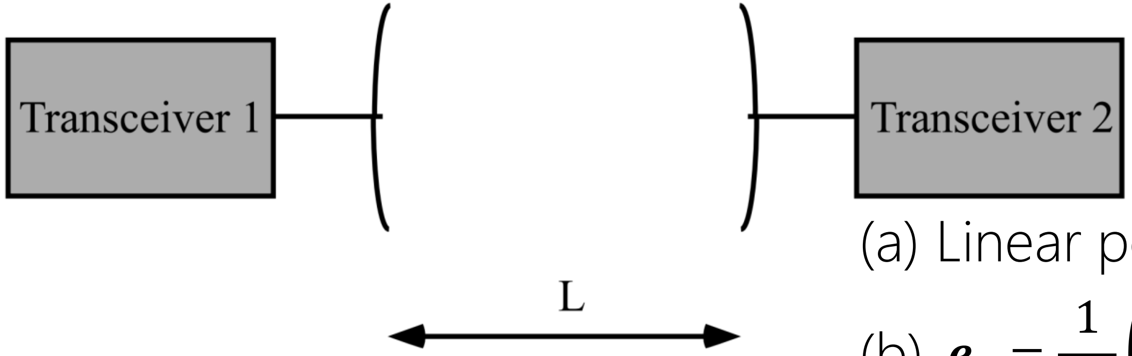
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

(a) Tx/Rx #2 linearly polarized options:

$$\boldsymbol{e}_2^{(1)} = \frac{1}{x}\begin{pmatrix}x\\ 0\\ 0\end{pmatrix} \qquad \boldsymbol{e}_2^{(2)} = \frac{1}{y}\begin{pmatrix}0\\ y\\ 0\end{pmatrix} \qquad \boxed{\boldsymbol{e}_2 = \frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}x\\ y\\ 0\end{pmatrix}}$$

Arbitrary orientation – tilted, but still linear!

$$\chi_{pol}^{(a)} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2 = \left|\frac{1}{\sqrt{2}}\frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ 0\end{pmatrix}\right|^2 = \frac{1}{2(x^2+y^2)}|x+jy+0|^2 = \frac{x^2+y^2}{2(x^2+y^2)} = 0.5$$

$$\chi_{pol,dB}^{(a)} = 10\log_{10} 0.5 = -3\ \text{dB}$$

(b)

$$\chi_{pol}^{(b)} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2 = \left|\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}\cdot\begin{pmatrix}1\\ 1-j\\ 0\end{pmatrix}\right|^2 = \frac{1}{6}|1+j+1|^2 = \frac{5}{6} \qquad \chi_{pol,dB}^{(b)} = 10\log_{10}\frac{5}{6} = -0.79\ \text{dB}$$

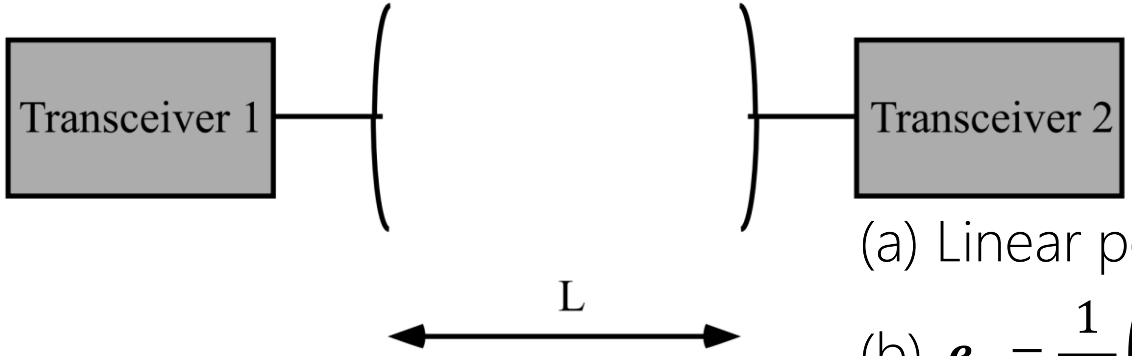
S11E01

- Depolarization factor?

$$\chi_{pol} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2$$

- Depolarization equal to 1 if antenna polarizations are matching perfectly

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}$$



(a) Linear polarization

(b) $\boldsymbol{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1+j\\ 0\end{pmatrix}$

(a) Tx/Rx #2 linearly polarized options:

$$\boldsymbol{e}_2^{(1)} = \frac{1}{x}\begin{pmatrix}x\\ 0\\ 0\end{pmatrix} \qquad \boldsymbol{e}_2^{(2)} = \frac{1}{y}\begin{pmatrix}0\\ y\\ 0\end{pmatrix} \qquad \boxed{\boldsymbol{e}_2 = \frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}x\\ y\\ 0\end{pmatrix}}$$

Arbitrary orientation – tilted, but still linear!

$$\chi_{pol}^{(a)} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2 = \left|\frac{1}{\sqrt{2}}\frac{1}{\sqrt{x^2+y^2}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ 0\end{pmatrix}\right|^2 = \frac{1}{2(x^2+y^2)}|x+jy+0|^2 = \frac{x^2+y^2}{2(x^2+y^2)} = 0.5$$

$$\chi_{pol,dB}^{(a)} = 10\log_{10} 0.5 = -3\text{ dB}$$

(b)

$$\chi_{pol}^{(b)} = |\boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*|^2 = \left|\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\begin{pmatrix}1\\ j\\ 0\end{pmatrix}\cdot\begin{pmatrix}1\\ 1-j\\ 0\end{pmatrix}\right|^2 = \frac{1}{6}|1+j+1|^2 = \frac{5}{6} \qquad \chi_{pol,dB}^{(b)} = 10\log_{10}\frac{5}{6} = -0.79\text{ dB}$$

* The conjugate serves to allign the reference directions

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122$ m,
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

(a) Antenna #2 gain: G_2=?

(b) ΔP_t=? ρ_1=ρ_2=-6 dB

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122\ \mathrm{m},$
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

  (a) Antenna #2 gain: G_2=?

  (b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

Friis formula:

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \underbrace{\chi_{pol}}_{\text{Depolarization}} \overbrace{(1-|\rho_1|^2)(1-|\rho_2|^2)}^{\text{Missmatch}}$$

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122$ m,
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

 (a) Antenna #2 gain: G_2=?

 (b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

Friis formula:

Missmatch

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \underline{\chi_{pol}} \overline{(1-|\rho_1|^2)(1-|\rho_2|^2)}$$

Depolarization

In dB:

$$10\log_{10} P_r = P_t + G_1 + G_2 - P_{loss} + \chi_{pol,dB} + \tau_{1,dB} + \tau_{2,dB}$$

$$P_{loss} = 10\log_{10}\left(\frac{4\pi L}{\lambda}\right)^2$$

$$\chi_{pol,dB} = 10\log_{10}\chi_{pol}$$

$$\tau_{dB} = 10\log_{10}(1-\rho^2) = 10\log_{10}\left(1-\left(10^{\frac{\rho_{dB}}{10}}\right)^2\right)$$

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122$ m,
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

(a) Antenna #2 gain: G_2=?

(b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

(a) Antennas matched:

$$\rho = 0, \rho_{dB} = -\infty \text{ dB} \Rightarrow \tau = 0 \text{ dB}$$

Friis formula:

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \underbrace{\chi_{pol}}_{\text{Depolarization}} \overbrace{(1-|\rho_1|^2)(1-|\rho_2|^2)}^{\text{Missmatch}}$$

In dB:

$$10\log_{10} P_r = P_t + G_1 + G_2 - P_{loss} + \chi_{pol,dB} + \tau_{1,dB} + \tau_{2,dB}$$

$$P_{loss} = 10\log_{10}\left(\frac{4\pi L}{\lambda}\right)^2$$

$$\chi_{pol,dB} = 10\log_{10}\chi_{pol}$$

$$\tau_{dB} = 10\log_{10}(1-\rho^2) = 10\log_{10}\left(1-\left(10^{\frac{\rho_{dB}}{10}}\right)^2\right)$$

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122\ \text{m},$
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

(a) Antenna #2 gain: G_2=?

(b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

Friis formula:

Missmatch

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \chi_{pol} (1-|\rho_1|^2)(1-|\rho_2|^2)$$

Depolarization

In dB:

$$10\log_{10} P_r = P_t + G_1 + G_2 - P_{loss} + \chi_{pol,dB} + \tau_{1,dB} + \tau_{2,dB}$$

$$P_{loss} = 10\log_{10}\left(\frac{4\pi L}{\lambda}\right)^2$$

$$\chi_{pol,dB} = 10\log_{10}\chi_{pol}$$

$$\tau_{dB} = 10\log_{10}(1-\rho^2) = 10\log_{10}\left(1-\left(10^{\frac{\rho_{dB}}{10}}\right)^2\right)$$

(a) Antennas matched:

$$\rho = 0, \rho_{dB} = -\infty\ \text{dB} \Rightarrow \tau = 0\ \text{dB}$$

$$G_2 = P_r - P_t + P_{loss} - \chi_{pol,dB} - G_1 = -100 - 10 + 114.2 - 3 + 0.46 = 1.66\ \text{dBi}$$ Gain (dBi)

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122\ \mathrm{m},$
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

(a) Antenna #2 gain: G_2=?

(b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

Friis formula:

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \underbrace{\chi_{pol}}_{\text{Depolarization}} \overbrace{(1-|\rho_1|^2)(1-|\rho_2|^2)}^{\text{Missmatch}}$$

In dB:

$$10\log_{10} P_r = P_t + G_1 + G_2 - P_{loss} + \chi_{pol,dB} + \tau_{1,dB} + \tau_{2,dB}$$

$$P_{loss} = 10\log_{10}\left(\frac{4\pi L}{\lambda}\right)^2$$

$$\chi_{pol,dB} = 10\log_{10}\chi_{pol}$$

$$\tau_{dB} = 10\log_{10}(1-\rho^2) = 10\log_{10}\left(1-\left(10^{\frac{\rho_{dB}}{10}}\right)^2\right)$$

(a) Antennas matched:

$$\rho = 0, \rho_{dB} = -\infty\ \mathrm{dB} \Rightarrow \tau = 0\ \mathrm{dB}$$

$$G_2 = P_r - P_t + P_{loss} - \chi_{pol,dB} - G_1 = -100 - 10 + 114.2 - 3 + 0.46 = 1.66\ \mathrm{dBi}$$ Gain (dBi)

(b) Antenna matching -6 dBi:

$$\tau_1 = \tau_2 = 1 - \left(10^{\frac{-6}{10}}\right)^2 = 0.937 = -0.28\ \mathrm{dB}$$

S11E02

- Distance: L=5000 m
- Frequency: 2.45 GHz => $\lambda = \frac{c_0}{f} = 0.122\ \mathrm{m},$
- Reception sensitivity (noise floor): P_2=-100 dBm
- Transmitted power: P_t=10 mW (=10 dBm)
- Antenna #1 gain: G_1=3 dBi
- Depolarization χ_{pol}=0.9

  (a) Antenna #2 gain: G_2=?

  (b) ΔP_t=? $\rho_1=\rho_2$=-6 dB

Friis formula:

Missmatch

$$P_r = P_t G_1 G_2 \left(\frac{\lambda}{4\pi L}\right)^2 \chi_{pol} (1-|\rho_1|^2)(1-|\rho_2|^2)$$

Depolarization

In dB:

$$10\log_{10} P_r = P_t + G_1 + G_2 - P_{loss} + \chi_{pol,dB} + \tau_{1,dB} + \tau_{2,dB}$$

$$P_{loss} = 10\log_{10}\left(\frac{4\pi L}{\lambda}\right)^2$$

$$\chi_{pol,dB} = 10\log_{10}\chi_{pol}$$

$$\tau_{dB} = 10\log_{10}(1-\rho^2) = 10\log_{10}\left(1-\left(10^{\frac{\rho_{dB}}{10}}\right)^2\right)$$

(a) Antennas matched:

$$\rho = 0, \rho_{dB} = -\infty\ \mathrm{dB} \Rightarrow \tau = 0\ \mathrm{dB}$$

$$G_2 = P_r - P_t + P_{loss} - \chi_{pol,dB} - G_1 = -100 - 10 + 114.2 - 3 + 0.46 = 1.66\ \mathrm{dBi}$$

Gain (dBi)

(b) Antenna matching -6 dBi:

$$\tau_1 = \tau_2 = 1 - \left(10^{\frac{-6}{10}}\right)^2 = 0.937 = -0.28\ \mathrm{dB}$$

$$P_t = P_r + P_{loss} - G_1 - G_2 - \chi_{pol,dB} - 2\tau_{dB} = 10.56\ \mathrm{dBm}$$

Power (dBm)